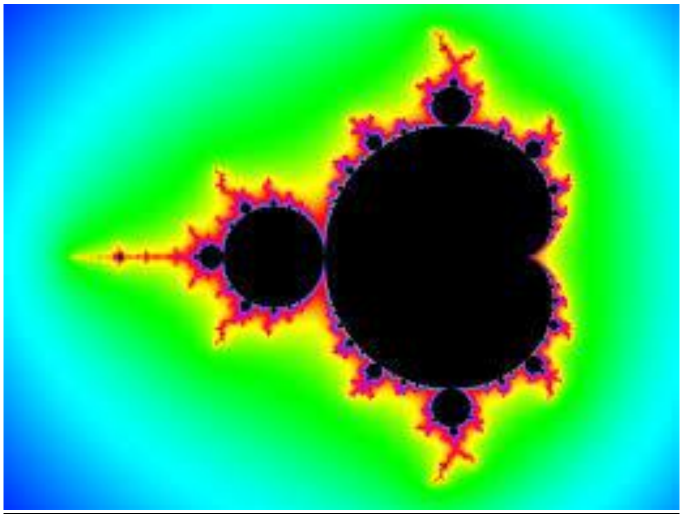
The Mandelbrot Set



Here is a video zooming in on the Mandelbrot Set
https://www.youtube.com/watch?v=PD2XgQOyCCk

Here is the first picture of it produced in 1978. As you will see it takes a lot of computing power to draw the set in any detail.

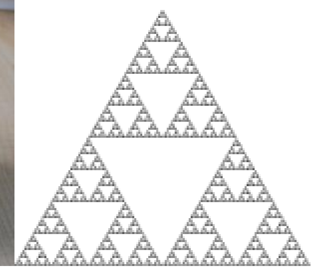
There is some disagreement over who discovered the set.
http://www.scientificamerican.com/article/mandelbrot-set-1990-horgan

It was named in honour of Benoit Mandelbrot, a mathematician working for IBM. He had earlier coined the term *fractal.* Fractals are things that show a repeating pattern at every scale. The Mandelbrot set is a fractal. Below are some examples. Below are some pictures of fractals.

Lightning Scar

Fern

Italian Broccoli



Sierpinski Triangle

The Mandelbrot Set is produced by an equation which is repeated (iterated). The equation is very simple, but first let's deal with *iteration*

$Z = Z^2 + c$ We start with Z = 0

let c = 0.1	0.2	-1	0.5	-0.5	2
0.1	0.2	-1	0.5	-0.5	2
0.11	0.24	0	0.75	-0.25	6
0.1121	0.2576	-1	1.0625	-0.4375	38
0.11256641	0.26635776	0	1.62890625	-0.30859375	1446
0.112671197	0.270946456	-1	3.153335571	-0.404769897	2090918
0.112694799	0.273411982	0	10.44352523	-0.33616133	4.37194E+12
0.112700	0.274754112	-1	109.5672191	-0.38699556	1.91138E+25
0.112701317	0.275489822	0	12005.47551	-0.350234436	3.65339E+50
0.112701587	0.275894642	-1	144131442.7	-0.37733584	1.3347E+101
0.112701648	0.276117854	0	2.07739E+16	-0.357617664	1.7815E+202
0.112701661	0.276241069	-1	4.31554E+32	-0.372109606	#NUM!
0.112701664	0.276309128	0	1.86239E+65	-0.361534441	#NUM!
0.112701665	0.276346734	-1	3.4685E+130	-0.369292848	#NUM!
bounded	bounded	bounded	blow up	bounded	blow up

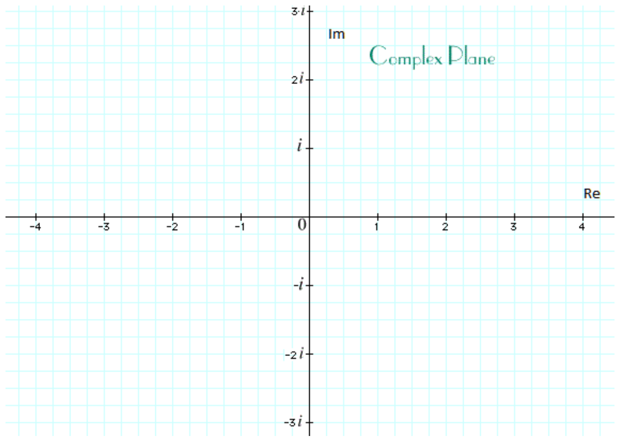
For any value of *c*, Z becomes 'bounded' or Z 'blows up' (escapes or goes to infinity)

The Mandelbrot Set is generated by iterating an equation that is very similar to the one above. The difference is that *c* is a complex number. Here is the equation:

$$Z_{(n+1)} = Z_{(n)} + c$$

That's all there is to it.

Here is a picture of the complex plane



And off we go: Pick a point *c* on the complex plane

Starting with Z = 0 iterate $Z = Z^2 + c$

Keep iterating until Z becomes bounded or Z blows up. Here is where the computer comes in handy.

Here are the first 15 iterations for c = -0.75 + 0.1i

c =	**-0.75+0.1i**
Iterate	**Recurrence Result**
z0	0
z1	-0.75+0.1i
z2	-0.1975-0.05i
z3	-0.71349375+0.11975i
z4	-0.255266731210938-0.070881753125i
z5	-0.689863118862956+0.136187506845439i
z6	-0.292635914253452-0.08790147644513i
z7	-0.67209089125028+0.151446257847498i
z8	-0.321229802914415-0.103571300826489i
z9	-0.657538428074457+0.166540377104165i
z10	-0.34537891281137-0.119013395544i
z11	-0.644877594904149+0.182209434325953i

z12	-0.367333165548024-0.135005563553932i
z13	-0.633292847678983+0.199184042053722i
z14	-0.388614451687503-0.152283658408824i
z15	-0.622169120557997+0.218359260827024i

Is Z going to be bounded or will it blow up?

How about?

z29	-0.109697447079467+0.85764566942738i
z30	-1.47352256439179-0.088163080869888i
z31	1.41349601894329+0.359820578016157i
z32	1.11850014720465+1.11720990911942i
z33	-0.747115401737799+2.59919889561713i
z34	-6.94765347546348-3.78380305419087i
z35	33.2027232622156+52.677104879837i

Look at that 33!

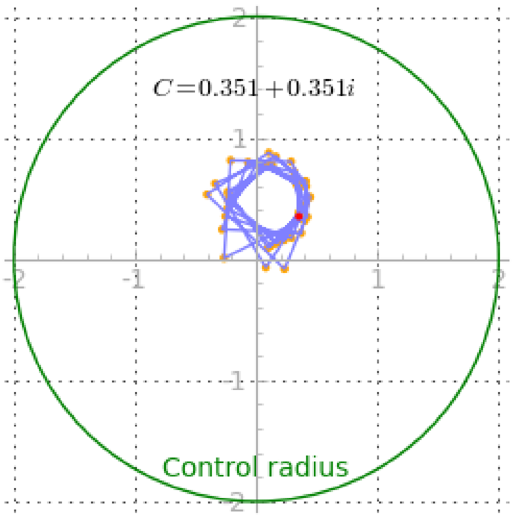
z38	-47971420674004.9+220953260345567i
z39	-4.65190860558536E+28-2.11988836026602E+28i
z40	1.71463270147278E+57+1.97230538120035E+57i
z41	-9.50023215752015E+113+6.76355860779371E+114i
z42	-4.48431809305926E+229-1.28510753970068E+229i
z43	#NUM!

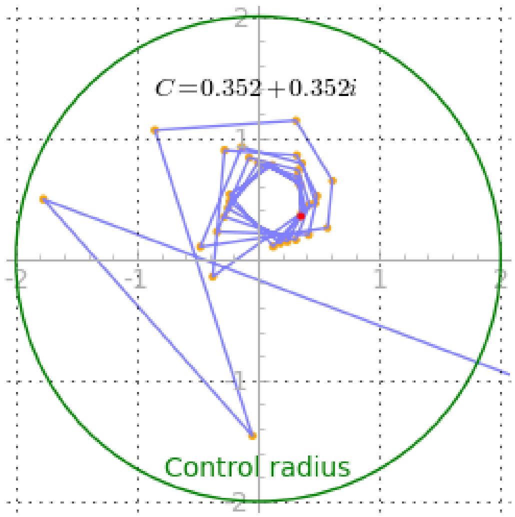
Blow up

Sometimes you have to let the iteration run for some time before you know whether Z will become bounded or blow up.

If Z becomes bounded, put a black dot on the point on the complex plane.
If Z blows up, don't do anything.

Now pick another point and repeat. That's it. You will have the Mandelbrot set. The more points you pick, the better. The diagrams below show what happens to Z drawn on the complex plane





Note that the two values of c are very close, but the outcomes are quite different. It takes 39 iterations before the c = 0.352 +0.352i escapes.

The *control radius* is 2.   If Z becomes 2, it will always eventually escape. The whole Mandelbrot Set exists within a circle of radius 2.

What about the colors? You keep count of how many iterations it takes for Z to escape. (Z > 2). Then you develop an algorithm. For example, if it takes between 20 and 30 iterations for Z to escape, colour the point red, if it takes between 31 and 40, colour the point **blue**. All the coloured regions are, of course, not part of the Mandelbrot set.

Many of the regions have names so people can refer to them

http://www.nahee.com/Derbyshire/manguide.html   Note that region1, 'seahorse valley' (c = -.75, 0.1i) is the region I chose as the example above.

http://fraqtive.mimec.org/downloads might interest you.

The Mandelbrot set is a funny thing: It's only interesting around the edges. There's a lesson there somewhere.

John Owen, April 2016