**Chem 100 2017** **Chaos**

**Chaos**: When the present determines the future, but the approximate present does not approximately determine the future.
or A ---------> B but ~A ---✖----> B

Chaos is not randomness

Randomness: The output is always random no matter what the input is.
Chaos: The same input gives the same output every time. But the inputs have to be *exactly* the same

In a forest community there are different populations, such as grass, rabbits, and foxes. The rabbits eat the grass. The foxes eat the rabbits. If there were no foxes, the rabbit population would grow too quickly and there wouldn't be enough grass for all of the rabbits to eat. Then the rabbits would begin to die, and the foxes would soon die because there is nothing but grass to eat.

We shall look a model of this. The *logistic map* is often cited as an archetypal example of how complex, chaotic behavior can arise from very simple non-linear dynamical equations.

The map was popularized in a seminal 1976 paper by the biologist Robert May.

$$x_{n+1} = r\, x_n\, (1 - x_n) \qquad \text{or} \qquad x_{new} = r\, x_{old}\, (1 - x_{old})$$

x is a number between zero and one, and represents the ratio of existing population to the maximum possible population at year n

r is a positive number, and represents a combined rate for reproduction and starvation.

This nonlinear difference equation is intended to capture two effects.

- reproduction where the population will increase at a rate proportional to the current population when the population size is small.

- starvation (density-dependent mortality) where the growth rate will decrease at a rate proportional to the value obtained by taking the theoretical " carrying capacity" of the environment less the current population.

The spreadsheet ‘Chaos’ runs the logistic map

$x_{start} = 0.1$
there are 50 cycles
There are 5 charts : r = 1, r = 2, r = 3, r = 4 and a composite of the first four.

The charts might surprise you

At r = ~ 3.56995 is the onset of chaos. Although the logistic map equation is absolutely deterministic (there is no randomness at all) once chaos is reached you cannot predict what the next number will be.

One of the features of chaos is that a slight difference in starting conditions leads to a big difference later on.

Edward Lorenz was a professor of meteorology at MIT. He became convinced that the way to go in the field was to use nonlinear methods (the output is not directly proportional to the input). He was repeatedly running a model through cycles and to save time and paper, he started at, for example, n = 50. The value of x was 0.506. To his surprise the output started to differ from the output he got when he started with n = 1.

This is shown in the spreadsheet, 'Lorenz'

He tracked down why there was a difference and found that, although the output showed x = 0.506, the computer stored the number as 0.506**127**. That this small difference caused such a huge difference led to birth of the 'Butterfly Effect'

*Does the flap of a butterfly's wings in Brazil set off a tornado in Texas?*

Here's a link to the Butterfly Effect

https://www.youtube.com/watch?v=aAJkLh76QnM

Chaos is everywhere: the double pendulum, a dripping tap, turbulence

Here is a video of a double pendulum

https://www.youtube.com/watch?v=InBBFF0j040

John Owen